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Brief Paper

A Design Scheme of Variable Structure Adaptive Control for Uncertain Dynamic Systems*

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Key Words—Variable structure control; adaptive control; uncertain dynamic systems; logical control; robust control.

Abstract—A design scheme of variable structure adaptive control for linear time-invariant system with uncertain dynamics is proposed. Both additive and multiplicative unmodeled dynamics are taken into consideration. The transfer function of the modeled part of the plant may have unstable zeros and unstable poles. A sign-following system and logic switchings are introduced into the control system. The global stability of the overall system is proved. Simulation results show the effectiveness of the proposed method.

1. *Introduction*

A model reference adaptive control (MRAC) scheme that can guarantee global asymptotic stability for systems without unstable zeros and unmodeled dynamics was proposed by Narendra and Valavari (1978) and Narendra *et al.* (1980). It is regarded as a landmark in the development of MRAC theory. However, Rohrs *et al.* (1985) showed that Narendra's MRAC scheme cannot always guarantee global stability if unmodeled dynamics and bounded external disturbances are present. Since then, the problem of robustness of MRAC has received considerable attention. Many attempts have been made to enhance the robustness of MRAC by counteracting the effects of unmodeled dynamics and external disturbance. Many modified MRAC algorithms have been proposed. Thus far the main achievements are as follows. An approach for handing bounded external disturbances requires a reference input signal that has a sufficient range of frequencies for the measurement vector to be persistently exciting in order to achieve the robustness of the controller (see e.g. Kosut and Johnson, 1984; Kokotovic *et al.* 1985; Anderson *et al.*, 1986; Narendra and Annaswamy, 1986, 1989; Sastry and Bodson, 1989). The dead-zone method was introduced by Egardt (1980) to make MRAC systems less sensitive to unmodeled dynamics. This method was further developed by Kreisselmeier and Narendra (1982), Peterson and Narendra (1982), Samson (1983), Sastry (1984), Kreisselmeier and Anderson (1986) and others. The σ-modification method was proposed and improved by Ioannou and Kokotovic (1984), Iannou (1986), Iannou and Taskalis (1986a), Ortega *et al.* (1987) and others. The idea of normalizing signals was introduced by Praly (1984, 1986), and was further studied and improved by Ioannou and K. S. Taskalis (1986b) and Tao and Ioannou (1991). Variable structure schemes were introduced into MRAC by Hsu and Costa (1989), Fu (1991, 1992) and Wu *et al.* (1992). These improvements simplify the ordinary MRAC scheme. All these works attempt to improve the robustness of Narendra's MRAC scheme by counteracting the unmodeled dynamics and external disturbances.

Narendra's MRAC scheme is generally used to deal with systems whose modeled part is of minimum phase. It is based upon the assumption that the adaptively tuned system can match perfectly with a reference model, i.e. there exists a procedure of controller parameterization that can make the closed-loop transfer function equal to the reference model in the absence of unmodeled dynamics. Unfortunately, for systems whose modeled part is of non-minimum phase, this requirement usually cannot be met. In general, the design of adaptive control for non-minimum-phase systems is full of trouble. In the above-mentioned literature, the minimum-phase condition is usually required for the modeled part of the plant.

Morse (1990, 1992) provided a unified theory of parameter adaptive control. He pointed out that, for a linear stationary process, a properly designed certainty equivalence control results in a tunable closed-loop parameterized system. As a result of tunability, the closed-loop parameterized system can be stabilized according to the tunability theorem and the theorem of certainty equivalence output stabilization proposed in his paper. Then, based on this theory Morse *et al.* (1992) proposed a hysteresis switching algorithm for the parameter adaptive control. It was shown that this algorithm is applicable to a large group of linear processes where the relative degree of their transfer functions and the sign of high-frequency gain may be unknown. As shown in Morse (1992), in order to ensure the global stability of the overall system, the closed-loop parameterized system should be tunable. It was shown (Morse *et al.*, 1992) that if the plant is a minimum-phase system, the closed-loop parameterized system will be 'tunable'. But, it is not clear whether a non-minimum-phase system or not is 'tunable'.

From the above discussion, we can see that the problem of adaptive control is still open for uncertain dynamic systems whose modeled part is of non-minimum phase.

Studies by Utkin (1977, 1987) have shown that variable structure control systems are insensitive to parameter perturbations and external disturbances. This inspires us to use logic switching to enhance the robustness of adaptive control. Feng (1986) introduced a sign-following system (SFS) into MRAC design. MRAC schemes can be further simplified by using SFS, and the robustness is greatly improved. This kind of SFS switching will also be used in the design scheme proposed in this paper.

In this paper, a design scheme of adaptive control with variable structure is presented for systems with unmodeled dynamics. The transfer function of the modeled part of the plant may have unstable zeros and unstable poles. The relative degree of the modeled part of the plant may be equal to or greater than one. The sign of the high-frequency gain may also be unknown. Thus the systems under study are uncertain dynamic systems in a very general sense. SFS and proper logic switchings are used in the control system to guarantee global stability.

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2. *System description*

Consider a single-input, single-output (SISO) linear time-invariant plant with additive and multiplicative unmodeled dynamics as described by the equation

$$y_0(t) = p(s)u(t) = p_0(s)[1 + \mu\Delta_1(s)]u(t) + \mu\Delta_2(s)u(t), \quad (1)$$

where $p(s)$ is strictly proper, $p_0(s)$ is the transfer function of the modeled part of the plant, $p_0(s) = k_0N_0(s)D_0^{-1}(s)$ and $\mu\Delta_1(s)$ and $\mu\Delta_2(s)$ are the additive and multiplicative perturbations respectively. Without loss of generality, let us assume that μ is a positive number. For this plant we shall make the following assumptions:

(A1) $p_0(s)$ is a strictly proper transfer function; $D_0(s)$ and $N_0(s)$ are monic polynomials of degrees n and m;

(A2) the system order n is known, but the relative degree $n^* = n - m$ and the sign of the high-frequency gain k_0 may be unknown;

(A3) $\Delta_1(s)$ is stable; it may be not proper if $n^* = n - m > 1$;

(A4) $\Delta_2(s)$ is a stable operator with relative degree greater than one;

(A5) a lower bound $\bar{q}_1 > 0$ on the stability margin $q_1 > 0$ for which the poles of $\Delta_1(s \times q_1)$ and $\Delta_2(s - q_1)$ are stable is known;

(A6) the impulse response functions $h_1(t)$ and $h_2(t)$ of $(s+q)^{-n^*+1}\Delta_1(s)$ and $(s+q)\Delta_2(s)$ satisfy the condition

$$\|h_i(t)\|_1 = \int_0^\infty |h_i(\tau)|\,d\tau \le K, \quad i = 1,2,$$

where $q > 0$, and K is a certain positive constant.

Remark 2.1. It is noteworthy that in the above assumptions, the modeled part $p_0(s)$ of the plant may have unstable zeros. This is different from the usual assumptions made for ordinary MRAC (i.e. in Narendra and Annaswamy, 1986; Ioannou and Tsakalis, 1986a, etc.).

Remark 2.2. Assumption (A3) implies that a small μ will lead to a small $|\mu\Delta_1(j\omega)|$ in the low-frequency range. However, since $\Delta_1(s)$ may be non-proper if $n^* = n - m > 1$, $|\mu\Delta_1(s)|$ may be large in the high-frequency range (Ioannou and Tsakalis, 1986a). In this paper, we shall assume that the relative degree of the transfer function $[N_0(s)/D_0(s)]\Delta_1(s)$ is greater than one.

In this paper, the variable structure adaptive control problem is briefly stated as follows. Given a reference model

$$y_m(t) = W_m(s)r(t) = \frac{k_m N_m(s)}{D_m(s)} r(t), \quad (2)$$

where $D_m(s)$ is a monic Hurwitz polynomial of degree n, $N_m(s)$ is a monic polynomial with degree less than n, and $r(t)$ is an arbitrary uniformly bounded and piecewisely-continuous external input signal. Design a suitable adaptive control law for the system (1) under the conditions (A1)–(A6), so that for some $\mu^* > 0$ and any $\mu \in [0, \mu^*)$, the overall system is stabilized and the plant output $y_0(t)$ will track the output $y_m(t)$ of the system (2) as closely as possible in spite of the existence of unmodeled dynamics $\Delta_1(s)$ and $\Delta_2(s)$ that satisfy assumptions (A3)–(A6).

In this paper, the basic control scheme is different from that in Narendra and Valavani (1978) and Narendra *et al.* (1980). The main difference between the usual MRAC schemes and ours is the use of a sign-following system and logic switchings. In the following sections, we shall present the design scheme of the variable structure adaptive control, and analyze its stability and performance.

3. *Design of variable structure adaptive controller*

The main points of the adaptive variable structure are as follows.

(i) An auxiliary error model is introduced. The general representation of the MRAC system proposed by Narendra and Valavani (1978) with an observer-type compensator is used to guarantee L_2 boundedness for the augmented error consisting of the auxiliary error and tracking error.

(ii) A sign-following system (SFS) is introduced, in which a minimum-phase model for sign comparison is used. Based on this minimum-phase model, the effect of non-minimum phase of the controlled plant can be extracted and used to improve our adaptive control algorithm. That is why a non-minimum-phase plant can be successfully controlled.

(iii) The control system is constructed in two hierarchies. First, a switching region is established, and variable structure sliding mode control is used outside this region to drive the state variables into the switching region. Secondly, inside this region, the normal variable structure adaptive control (VSAC) with a SFS is adopted.

Now let us explain the design of the controller in detail.

It will be more convenient for our design if the relative degree of the modeled part of the plant to be controlled is equal to one. If the relative degree is greater than one, we can use the following operations to make it equal to one. Let us define

$$u(t) = \frac{1}{s+\beta}\bar{u}(t), \quad (3)$$

where β is a proper positive constant. Transform the model (1)–(3) to the form

$$y_0(t) = \frac{k_0N_0(s)}{(s+\beta)D_0(s)}[1 + \mu\Delta_1(s)]\bar{u}(t) + \mu\Delta_2(s)\frac{1}{s+\beta}\bar{u}(t). \quad (4)$$

Define

$$y_1(t) = \frac{1}{s+\beta}\bar{u}(t), \quad (5)$$

$$y(t) = y_0(t) + y_1(t). \quad (6)$$

Then, from (4)–(6),

$$y(t) = \frac{k_0N_0(s) + D_0(s)}{(s+\beta)D_0(s)}[1 + \mu\Delta_1(s)]\bar{u}(t) + \mu\Delta_2(s)\frac{1}{s+\beta}\bar{u}(t) - \mu\frac{1}{s+\beta}\Delta_1(s)\bar{u}(t). \quad (7)$$

Now the modeled part in (7) is $[k_0N_0(s) + D_0(s)]/[(s+\beta)D_0(s)]$. Its relative degree and gain are equal to one. If $\bar{u}(t)$ and $y(t)$ are guaranteed to be bounded then, from (5) and (6), we know that $y_1(t)$ and $y_0(t)$ are also bounded.

From the above discussion, without loss of generality, we can still use (1) by assuming that the relative degree of $N_0(s)/D_0(s)$ is equal to 1 and that $k_0 = 1$. If the global stability is guaranteed for the system (7) then, from (4) and (5), we can conclude that the system (4) is also globally stable.

Let us first deduce the error equation. Define

$$R(s) \triangleq D_m(s) - D_0(s). \quad (8)$$

Since $D_m(s)$ and $D_0(s)$ are monic polynomials of order n,

$R(s)$ is a polynomial of order at most $n-1$. Substituting (8) into (1), we have

$$y_0(t)=\frac{N_0(s)}{D_m(s)}u(t)+\frac{R(s)}{D_m(s)}y_0(t)+\frac{\mu N_0(s)}{D_m(s)}\Delta_1(s)u(t)+\frac{\mu D_0(s)}{D_m(s)}\Delta_2(s)u(t). \quad (9)$$

Operating both sides of (9) with $s+\alpha$, we have

$$\dot{y}_0(t)=-\alpha y_0(t)+\frac{(s+\alpha)N_0(s)}{D_m(s)}u(t)+\frac{(s+\alpha)R(s)}{D_m(s)}y_0(t)+\frac{\mu(s+\alpha)N_0(s)}{D_m(s)}\Delta_1(s)u(t)+\frac{\mu D_0(s)}{D_m(s)}(s+\alpha)\Delta_2(s)u(t), \quad (10)$$

where α is a proper positive constant. Denote

$$d(t)\triangleq\frac{\mu(s+\alpha)N_0(s)}{D_m(s)}\Delta_1(s)u(t)+\frac{\mu D_0(s)}{D_m(s)}(s+\alpha)\Delta_2(s)u(t). \quad (11)$$

Then (10) is simplified as follows:

$$\dot{y}_0(t)=-\alpha y_0(t)+\frac{(s+\alpha)N_0(s)}{D_m(s)}u(t)+\frac{(s+\alpha)R(s)}{D_m(s)}\times y_0(t)+d(t). \quad (12)$$

Since $N_0(s)$ is assumed to be a monic polynomial of order $n-1$, we obtain

$$\frac{(s+\alpha)N_0(s)}{D_m(s)}u(t)=u(t)+\frac{(s+\alpha)N_0(s)-D_m(s)}{D_m(s)}u(t),$$

where $[(s+\alpha)N_0(s)-D_m(s)]/D_m(s)$ is still strictly proper. We can write $\{[(s+\alpha)N_0(s)-D_m(s)]/D_m(s)\}u(t)$ and $[(s+\alpha)R(s)/D_m(s)]y_0(t)$ in their state-space realizations as

$$\dot{x}=Ax+bu(t),$$
$$\dot{z}=Az+by_0(t),$$
$$\frac{(s+\alpha)N_0(s)-D_m(s)}{D_m(s)}u(t)=\theta_1^{\mathrm{T}}x(t), \quad (13)$$
$$\frac{(s+\alpha)R(s)}{D_m(s)}y_0(t)=\theta_2^{\mathrm{T}}z(t)+\theta_3y_0(t),$$

$$A=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\-\alpha_1&-\alpha_2&-\alpha_3&\cdots&-\alpha_n\end{bmatrix} \quad (14)$$

$$b=[0\ \ 0\ \ \ldots\ \ 0\ \ 2]^{\mathrm{T}}.$$

Here the α_i' $(i=1,2,\ldots,n)$ are the coefficients of the known polynomial $D_m(s)=s^n+\alpha_ns^{n-1}+\ldots+\alpha_1$. θ_1 is formed by the coefficients of the polynomial $\frac{1}{2}[(s+\alpha)N_0(s)-D_m(s)]$. θ_2 is formed by the coefficients of the polynomial $\frac{1}{2}[(s+\alpha)R(s)-\theta_3D_m(s)]$, where θ_3 is the coefficient of the term s^{n-1} in $R(s)$. θ_1, θ_2 and θ_3 are unknown, but $x(t)$ and $z(t)$ are measurable. Substituting (13) into (12), we obtain

$$\dot{y}_0(t)=-\alpha y_0(t)+u(t)+\theta_1^{\mathrm{T}}x(t)+\theta_2^{\mathrm{T}}z(t)+\theta_3y_0(t)+d(t). \quad (15)$$

Define the tracking error as

$$e_0(t)\triangleq y_0(t)-y_m(t). \quad (16)$$

From (2) and (16), the following equation for the error $e_0(t)$ can be obtained:

$$\dot{e}_0(t)=-\alpha e_1(t)+u(t)+\theta_1^{\mathrm{T}}x(t)+\theta_2^{\mathrm{T}}z(t)+\theta_3y_0(t)-(s+\alpha)W_m(s)r(t)+d(t). \quad (17)$$

In the sequel, an auxiliary error signal $e_1(t)$ is introduced, satisfying the equation

$$\dot{e}_1(t)=-\alpha e_1(t)+v(t), \quad (18)$$

where $v(t)$ will be defined later. Define an augmented error

$$\bar{e}_0(t)\triangleq e_1(t)-e_0(t), \quad (19)$$

and choose $v(t)$ to make $\bar{e}_0(t)$ as small as possible. Therefore we may take

$$v(t)=u(t)+\theta_1^{\mathrm{T}}(t)x(t)+\theta_2^{\mathrm{T}}(t)z(t)+\theta_3(t)y_0(t)-\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t)-(s+\alpha)W_m(s)r(t), \quad (20)$$

where $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$ are adjusted according to the rules

$$\dot{\theta}_1(t)=-\Gamma_1x(t)\bar{e}_0(t),$$
$$\dot{\theta}_2(t)=-\Gamma_2z(t)\bar{e}_0(t), \quad (21)$$
$$\dot{\theta}_3(t)=-\gamma_3y_0(t)\bar{e}_0(t),$$

where Γ_1 and Γ_2 are positive-definite matrices and γ_3 is a positive constant. $\bar{\mu}$ is an appropriate small positive constant, and $m(t)$ is a normalized signal obtained from the control signal $u(t)$ in the form

$$\dot{m}(t)=-\sigma m(t)+|u(t)|,\quad m(0)>0, \quad (22)$$

where $0<\sigma<\min\{q,|\operatorname{Re}\lambda_j(D_m(s))|\}$. It was proved by Ioannou and Tsakalis (1986a) that there exists a positive constant μ^* such that when $0\le\mu\le\mu^*$ the following inequality holds:

$$|d(t)|<\bar{\mu}m(t). \quad (23)$$

Denote $\tilde{\theta}_i(t)\triangleq\theta_i(t)-\theta_i$, $i=1,2,3$. Then, from (17)–(20),

$$\dot{\bar{e}}_1(t)=-\alpha\bar{e}_0(t)+\tilde{\theta}_1^{\mathrm{T}}(t)x(t)+\tilde{\theta}_2^{\mathrm{T}}(t)z(t)+\tilde{\theta}_3(t)y_0(t)-\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t)-d(t). \quad (24)$$

Now if $\bar{e}_0(t)$ is bounded and small and the control system is designed using (18)–(20) so that $e_1(t)$ is bounded and sufficiently small then the tracking error $e_0(t)$ will also be sufficiently small and bounded. Actually, we can prove that $\bar{e}_0(t)$ is small. Take the following positive definite function as Liapunov function:

$$V_1(t)=\tfrac{1}{2}\bar{e}_0^2(t)+\tfrac{1}{2}\tilde{\theta}_1^{\mathrm{T}}(t)\Gamma_1^{-1}\tilde{\theta}_1(t)+\tfrac{1}{2}\tilde{\theta}_2^{\mathrm{T}}(t)\Gamma_2^{-1}\tilde{\theta}_2(t)+\tfrac{1}{2}\tilde{\theta}_3^2\gamma_3^{-1}. \quad (25)$$

Then, from (23) and (24),

$$\dot{V}_1(t)\le-\alpha\bar{e}_0^2(t). \quad (26)$$

Integration of (26) yields

$$\alpha\int_0^t\bar{e}_0^2(\tau)\,d\tau+V_1(t)\le V_1(0).$$

This means $\bar{e}_0(t)\in L_2$. From the above discussion, we see that if $e_1(t)$ is actually bounded and small then the tracking error will be bounded and small. Therefore in the rest of this paper, we can use $e_1(t)$ defined by (18) instead of the tracking error $e_0(t)$ in the design of control laws.

Substituting (20) into (18), we have

$$\dot{e}_1(t)=-\alpha e_1(t)+u(t)+\theta_1^{\mathrm{T}}(t)x(t)+\theta_2^{\mathrm{T}}(t)z(t)+\theta_3(t)y_0(t)-\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t)-(s+\alpha)W_m(s)r(t). \quad (27)$$

In (13),

$$\dot{x}=Ax+bu(t),$$
$$\dot{z}=Az+by_0(t). \quad (28)$$

Now (27) together with (28) form a time-varying nonlinear system. Its parameters and signals are measurable. They will be used for the design of a variable structure adaptive controller.

The control system is constructed in two hierarchies. First, a variable structure sliding-mode control is used to drive the system state variables $x(t)$ and $e_1(t)$ into a band region Ω. Then, inside the band region, another variable structure adaptive control law with SFS is proposed to make the system state variables $x(t)$ and $e_1(t)$ bounded and small. Detailed explanations of the SFS will be given later.

Take the switching surface as

$$S = c^{\mathrm{T}}x + e_1, \tag{29}$$

where $c^{\mathrm{T}} = [c_1 \ c_2 \ \dots \ c_{n-1} \ 1]^{\mathrm{T}}$ is formed from the coefficients of monic Hurwitz polynomial $C(s) = s^{n-1} + c_{n-1}s^{n-2} + \dots + c_1$. Take a border region Ω in the neighborhood of the switching surface $S = 0$ with width δ, i.e. $|S| \le \delta$, where δ is a small positive constant. Let us first explain how the variable structure control to be used outside of the band region is designed. In this case, $|S| > \delta$. Take

$$\dot{S} = -k_1 \operatorname{sgn} S - k_2 S, \tag{30}$$

where k_1 and k_2 are positive. We have $S\dot{S} = -k_1|S| - k_2S^2$. This means that the sliding surface is reachable (see Utkin, 1977), and $x(t)$ and $e_1(t)$ will reach the border region Ω in finite time. From (27) and (28),

$$\begin{aligned}\dot{S} = {} & c^{\mathrm{T}}Ax(t) + 3u(t) - \alpha e_1(t) + \theta_1^{\mathrm{T}}(t)x(t) + \theta_2^{\mathrm{T}}(t)z(t) + \theta_3(t)y_0(t) \\ & - \bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]m(t) - (s+\alpha)W_{\mathrm{m}}(s)r(t).\end{aligned}$$

From (30), the sliding mode control is taken to be

$$\begin{aligned}u(t) = {} & \tfrac{1}{3}\{-c^{\mathrm{T}}Ax(t) + \alpha e_1(t) - \theta_1^{\mathrm{T}}(t)x(t) - \theta_2^{\mathrm{T}}(t)z(t) - \theta_3(t)y_0(t) \\ & + \bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t) + (s+\alpha)W_{\mathrm{m}}(s)r(t) \\ & - k_1 \operatorname{sgn} S - k_2 S\}.\end{aligned} \tag{31}$$

According to the selection of this control law, we can be sure that once $x(t)$ and $e_1(t)$ enter the region Ω, they will stay inside.

Next, let us study how the controller law is designed inside the border region Ω. The control process inside the region Ω is rather complicated. The main point of this scheme is the introduction of SFS. A minimum-phase model $C(s)/D_{\mathrm{m}}(s)$ is used for sign comparison. The sign of the output of this model, $[C(s)/D_{\mathrm{m}}(s)]u(t) = c^{\mathrm{T}}x(t)$, is compared with the sign of the error $e_1(t)$, and a logic switching function $\phi(t)$ is formed and introduced into the control.

Take $\phi(t_0) = \operatorname{sgn}[e_1(t_0)c^{\mathrm{T}}x(t_0)]$ at the initial instant t_0 if $e_1(t_0)c^{\mathrm{T}}x(t_0) \ne 0$. If $e_1(t_0)c^{\mathrm{T}}x(t_0) = 0$ then take $\phi(t_0) = 1$. Assume that t_k $(k = 0, 1, 2, \dots)$ is the switching instant, and take $\phi(t_k) = \operatorname{sgn}[e_1(t_k)c^{\mathrm{T}}x(t_k)]$. Define a piecewise-continuous function

$$\varepsilon(t) = \begin{cases} \eta(t) & \text{if } \eta(t) \le \varepsilon_0, \\ \varepsilon_0 & \text{if } \eta(t) > \varepsilon_0, \end{cases}$$

where

$$\eta(t) = \tfrac{1}{4}\alpha \int_{t_k}^{t} |c^{\mathrm{T}}x(\tau) + \phi(\tau)e_1(\tau)|\,\mathrm{d}\tau, \tag{32}$$

and ε_0 is a properly small positive constant. When $t > t_k$, if $|e_1(t)| < \varepsilon(t)$ or $|c^{\mathrm{T}}x(t)| < \varepsilon(t)$, we take $\phi(t) = \phi(t_k)$. If, at a certain time instant t'_{k+1}, we have $|e_1(t'_{k+1})| \ge \varepsilon(t'_{k+1})$ and $|c^{\mathrm{T}}x(t'_{k+1})| \ge \varepsilon(t'_{k+1})$ and $\operatorname{sgn}[e_1(t'_{k+1})c^{\mathrm{T}}x(t'_{k+1})] = -\phi(t_k)$ then $\phi(t)$ changes sign at t'_{k+1}. Define $t_{k+1} = t'_{k+1}$; then $\phi(t_{k+1}) = -\phi(t_k)$ and $\phi(t) = \phi(t_k)$ when $t \in [t_k, t_{k+1})$. When $t \ge t_{k+1}$, the above procedure is repeated.

The logic switching defined above is a kind of logic switching with a variable hysteresis. This hysteresis depends on the magnitude of the error $e_1(t)$, $x(t)$ and the chosen value of ε_0. The smaller is ε_0, the faster $\phi(t)$ changes its sign, and it will be more effective for reducing the tracking error. But the frequency of chattering will be higher. The purpose of the defined $\phi(t)$ is to make the sign of the output of the SFS follow the sign of the tracking error with sufficient speed; meanwhile, stiff switching is avoided so that the frequency of the chattering may be reduced.

Next, let us explain how to design the control law.

Define

$$\bar{x}_n(t) \triangleq x_n(t) + \phi(t)e_1(t), \tag{33}$$

here $x_n(t)$ is the nth component of the state variable $x(t)$. In the interval $[t_k, t_{k+1})$, from (27) and (28), we have

$$\begin{aligned}\dot{x}_1 &= x_2(t), \\ \dot{x}_2 &= x_3(t), \\ &\ \vdots \\ \dot{x}_{n-1} &= \bar{x}_n(t) - \phi(t)e_1(t), \\ \dot{\bar{x}}_n(t) &= -\alpha_1x_1(t) - \dots - \alpha_n\bar{x}_n(t) + (\alpha_n - \alpha)\phi(t)e_1(t) \\ &\quad + [2 + \phi(t)]u(t) + \phi(t)\theta_1^{\mathrm{T}}(t)x(t) + \phi(t)\theta_2^{\mathrm{T}}(t)z(t) \\ &\quad + \phi(t)\theta_3(t)u_0(t) - \phi(t)\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t) \\ &\quad - \phi(t)(s+\alpha)W_{\mathrm{m}}(s)r(t).\end{aligned} \tag{34}$$

Denote $\bar{x}(t) = [x_1(t) \ \dots \ x_{n-1}(t) \ \bar{x}_n(t)]^{\mathrm{T}} \triangleq [\bar{x}_1^{\mathrm{T}}(t) \ \bar{x}_n(t)]^{\mathrm{T}}$. To make a state-space transformation for the system (34), let us define

$$[\bar{x}_1^{\mathrm{T}}(t) \quad e_n(t)]^{\mathrm{T}} \triangleq T\bar{x}(t), \tag{35}$$

where the matrix T is

$$T = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & \dots & c_{n-1} & 1 \end{bmatrix}.$$

In the new coordinate system, we have

$$\begin{aligned}\dot{x}_1 &= x_2(t), \\ \dot{x}_2 &= x_3(t), \\ &\ \vdots \\ \dot{x}_{n-1} &= -c_1x_1(t) - \dots - c_{n-1}x_{n-1}(t) + e_n(t) - \phi(t)e_1(t),\end{aligned} \tag{36}$$

$$\begin{aligned}\dot{e}_n(t) = {} & a_{nn}e_n(t) + [2 + \phi(t)]u(t) \\ & + A_{10}\bar{x}_1(t) + (\alpha_n - \alpha - c_{n-1})\phi(t)e_1(t) \\ & + \phi(t)\theta_1^{\mathrm{T}}(t)x(t) + \phi(t)\theta_2^{\mathrm{T}}(t)z(t) + \phi(t)\theta_3(t)y_0(t) \\ & - \phi(t)(s+\alpha)W_{\mathrm{m}}(s)r(t) - \phi(t)\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]m(t),\end{aligned} \tag{37}$$

where $A_{10} \triangleq [\beta_1 \ \beta_2 \ \dots \ \beta_{n-1}]^{\mathrm{T}}$. Here β_i $(i = 1, 2, \dots, n-1)$ and a_{nn} are obtained from the calculation by using (34) and (35). They are constant and known, since T and c are known, and they are not related to α. The polynomial $C(s) = s^{n-1} + c_{n-1}s^{n-2} + \dots + c_1$ is Hurwitzian. Hence, from (36), it is easy to show that boundedness of $e_n(t)$ and $e_1(t)$ leads to boundedness of $x_1(t), x_2(t), \dots, x_{n-1}(t)$ in (36). Meanwhlile from (33) and (35)we have

$$e_n(t) = c^{\mathrm{T}}x(t) + \phi(t)e_1(t). \tag{38}$$

According to the definition of $\phi(t)$, the boundedness of $e_n(t)$ leads to the boundedness of $c^{\mathrm{T}}x(t)$ and $e_1(t)$. Therefore we have to select $u(t)$ in (37) to make $e_n(t)$ bounded. Let us define $u(t)$ as

$$\begin{aligned}u(t) = {} & \frac{1}{2+\phi(t)}\{-A_{10}\bar{x}_1(t) - (\alpha_n - \alpha - c_{n-1})\phi(t)e_1(t) \\ & - \phi(t)\theta_1^{\mathrm{T}}(t)x(t) - \phi(t)\theta_2^{\mathrm{T}}(t)z(t) - \phi(t)\theta_3(t)y_0(t) \\ & + \phi(t)(s+\alpha)W_{\mathrm{m}}(s)r(t) + \phi(t)\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]m(t) \\ & - \alpha e_n(t) - k_1 \operatorname{sgn}[e_n(t)]\}.\end{aligned} \tag{39}$$

Here a_{nn}, A_{10}, α_n, α, $\bar{\mu}$ and k_1 are known. $\phi(t)$, $\theta_1(t)$, $\theta_2(t)$, $\theta_3(t)$, $m(t)$, $r(t)$, $e_1(t)$, $y_0(t)$, $\bar{x}_1(t)$, $e_n(t)$ and $\bar{e}_0(t)$ are measurable. Substituting (39) into (37), we have

$$\dot{e}_n(t) = -(\alpha + a_{nn})e_n(t) - k_1 \operatorname{sgn}[e_n(t)]. \tag{40}$$

Since a_{nn} is not related to α, we can select α such that $\alpha + a_{nn} > \frac{1}{2}\alpha$. This means that $e_n(t)$ will decay exponentially.

Now the design of the VSAC with SFS is complete.

In this section, we have discussed the controller design as a whole. We shall proceed to analyse closed-loop stability with our control laws.

4. *Stability analysis*

First let us show that $u(t)$ is bounded when the state variable $x(t)$ and $e_1(t)$ are outside the border region Ω. From (31) and (39), we know that $u(t)$ is not a continuous function. But, from (27) and (28), we know that $e_1(t)$ and $c^Tx(t)$ are continuous. Beginning with $t=0$, let us assume that, after t_0, the $e_1(t)$ and $x(t)$ are forced into the region Ω using the control law defined by (31). Since t_0 is limited, $x(t)$, $e_1(t)$ and $y_0(t)$ in $[0, t_0)$ are bounded. Since $\theta_i(t)$ $(i=1,2,3)$ and $r(t)$ are bounded, from (31), we can rewrite (31) as

$$\begin{aligned} u(t) &\triangleq \tfrac{1}{3}\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t) + g(t), \\ g(t) &= \tfrac{1}{3}[-c^TAx(t) + \alpha e_1(t) - \theta_1^T(t)x(t) - \theta_2^T(t)z(t) - \theta_3(t)y_0(t) \\ &\quad + (s+\alpha)W_m(s)r(t) - k_1\operatorname{sgn} S - k_2 S], \end{aligned} \tag{41}$$

where $g(t)$ is bounded for $t \in [0, t_0)$. From (22), we have

$$\begin{aligned} \dot{m}(t) &= -\sigma m(t) + |\tfrac{1}{3}\bar{\mu}\operatorname{sgn}[\bar{e}_0(t)]\,m(t) + g(t) \\ &\le (-\sigma + \tfrac{1}{3}\bar{\mu})m(t) + |g(t)|. \end{aligned} \tag{42}$$

Since $\sigma > 0$ and $\bar{\mu}$ is sufficiently small, we take $-\sigma + \frac{1}{3}\bar{\mu} < 0$. Therefore, from (42), we know that $m(t)$ is bounded. Finally, we conclude from (41) that $u(t)$ is bounded in the interval $[0, t_0)$.

Now we shall reform a stability analysis of the system when $x(t)$ and $e_1(t)$ are within the border region Ω. First, let us consider the case where the time interval $[t_k, t_{k+1})$ $(k = 0, 1, 2, \ldots)$ is not zero. We then have $\cup [t_k, t_{k+1}) = [t_0, \infty)$. In the interval $[t_k, t_{k+1})$, $\phi(t)$ remains either 1 or -1. For the interval $[t_k, t_{k+1})$, we shall take the Liapunov function as

$$V(e_n(t)) = |e_n(t)| = |c^Tx(t) + \phi(t)e_1(t)|. \tag{43}$$

Since the sign of $\phi(t)$ does not change in $[t_k, t_{k+1})$, $c^Tx(t) + e_1(t)$ is differentiable. In the interval $[t_k, t_{k+1})$, from (40), we have

$$\dot{V}(e_n(t)) = -(\alpha + a_{nn})\,|e_n(t)| - k_1, \quad t \in [t_k, t_{k+1}). \tag{44}$$

Here, the points where $V(e_n(t)) = 0$, the differentiation may be taken from left or right. That is to say, in the interval $[t_k, t_{k+1})$, we have

$$V(e_n(t)) - V(e_n(t_k)) \le -\tfrac{1}{2}\alpha \int_{t_k}^{t} |e_n(\tau)|\,d\tau.$$

Defining $V(e_n(t_{k+1} - 0) \triangleq \lim_{t \to t_{k+1}} V(e_n(t))$ $(t_k < t < t_{k+1})$, we have

$$V(e_n(t_{k+1} - 0)) + \tfrac{1}{2}\alpha \int_{t_k}^{t_{k+1}} |e_n(\tau)|\,d\tau \le V(e_n(t_k)). \tag{45}$$

This implies that $V(e_n(t))$ has been reduced by $\frac{1}{2}\alpha \int_{t_k}^{t_{k+1}} |e_n(\tau)|\,d\tau$ at least during the interval $[t_k, t_{k+1})$. According to the above definition of the switching function $\phi(t)$, $V(e_n(t))$ will have a maximum possible jump of $\varepsilon(t_{k+1}) \le \frac{1}{2}\alpha \int_{t_k}^{t_{k+1}} |e_n(\tau)|\,d\tau$ at t_{k+1}. From (45), we have

$$\begin{aligned} &V(e_n(t_{k+1} - 0)) + \varepsilon(t_{k+1}) \le V(e_n(t_k)), \\ &V(e_n(t_{k+1})) \le V(e_n(t_k)) \ (k = 0, 1, 2, \ldots). \end{aligned} \tag{46}$$

The curve of $V(t)$ is depicted in Fig. 1.

From the above analysis, we can see that $e_n(t)$ is bounded in the interval $[t_0, \infty)$. From the definitions of $e_n(t)$ and $\phi(t)$, we have

$$|c^Tx(t)| \le |e_n(t)| + \varepsilon_0, \quad |e_1(t)| \le |e_n(t)| + \varepsilon_0. \tag{47}$$

Hence the boundedness of $e_1(t)$ and $c^Tx(t)$ is obtained from the boundedness of $e_n(t)$. According to (36) and (37), the boundedness of $e_1(t)$ and $e_n(t)$ implies that $\bar{x}_1(t)$, as well as $x_n(t)$, is also bounded. With a similar argument for (41), the control signal within the region of Ω can be written as

$$u(t) = \frac{1}{2 + \phi(t)}\bar{\mu}\phi(t)\operatorname{sgn}[\bar{e}_0(t)]\,m(t) + f(t), \tag{48}$$

where $f(t)$ is a certain bounded function obtained from (39). By an analysis similar to that for $|S| > \delta$, we can show that $u(t)$ is also bounded in the case $|S| < \delta$.

If the interval $[t_j, t_{j+1})$ approaches zero for a certain number j, and $\phi(t)$ changes its sign at an extremely high rate, we shall show that our control system can still work well. Actually, by the definition of $\phi(t)$, we have $c^Tx(t_j) = 0$ or $e_1(t_j) = 0$. But, in the region Ω, we have

$$|e_1(t) + c^Tx(t)| < \delta. \tag{49}$$

Now, if high-frequency chattering occurs at t_j then $e_1(t_j) = 0$ or $c^Tx(t_j) = 0$. Hence $|e_1(t_j)| < \delta$ and $|c^Tx(t_j)| < \delta$ can still be ensured. Furthermore, from (49), we have

$$|e_1(t)| < 2\delta, \quad |c^Tx(t)| < 2\delta, \quad t \in [t_j, t_{j+1}). \tag{50}$$

That means that $e_1(t)$ and $c^Tx(t)$ remain bounded, even though high-frequency chattering happens. From (36), we can see that boundedness of $c^Tx(t)$ leads to boundedness of $x_1(t), x_2(t), \ldots, x_{n-1}(t)$. $y_0(t)$ is bounded if $e_1(t)$ is bounded. Finally, we can conclude that $u(t)$ defined by (39) is always bounded.

Now let us analyse the motion along the switching surface $|S| = \delta$ in detail. In Fig. 2, we show the possible path of motion along the boundary of region Ω. At instant $\bar{t}_0 > 0$, the moving point reaches the boundary of Ω. Without loss of generality, we assume $S = \delta$. Then $c^Tx(\bar{t}_0) + e_1(\bar{t}_0) = \delta$. In the region $S > 0$, we have $\dot{S} > 0$. i.e.

$$\dot{e}_1(t) + c^T\dot{x}(t) < 0.$$

This shows that $e_1(t)$ and $c^Tx(t)$ cannot enter the region $|S| > \delta$.

Since in the region Ω, (40) is valid and $\phi(t) = -1$, as shown in Fig. 2, we have

$$\begin{aligned} \dot{e}_n(t) &= c^T\dot{x}(t) - \dot{e}_1(t) \\ &= -\alpha c^Tx(t) + \alpha e_1(t) - k_1\operatorname{sgn}[e_n(t)] > 0; \end{aligned} \tag{51}$$

that is,

$$c^T\dot{x}(t) > \dot{e}_1(t) - k_1\operatorname{sgn}[e_n(t)] > \dot{e}_1(t). \tag{52}$$

This shows that the change of $c^Tx(t)$ is faster than that of $e_1(t)$. Therefore $c^Tx(t)$ and $e_1(t)$ can move along the solid line and not along the dotted arrows, as shown in Fig. 2. In fact, suppose the moving point did move along the dotted arrows. Then $c^T\dot{x}(t) < \dot{e}_1(t) < 0$ or $c^T\dot{x}(t) < 0 < \dot{e}_1(t)$. This contradicts the form (52). From (30) and (40), we shall have

$$|c^Tx(t)| + |e_1(t)| < |c^Tx(\bar{t}_0)| + |e_1(\bar{t}_0)|;$$

hence the moving point tends to go to the origin. The state variables $x(t)$ and $e_1(t)$ are always bounded. Boundedness of the state variables implies boundedness of the control signal. The above analysis shows that once the moving point enters

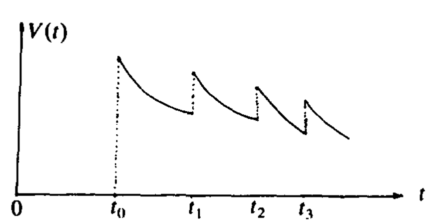

Fig. 1. Liapunov function $V(t)$.

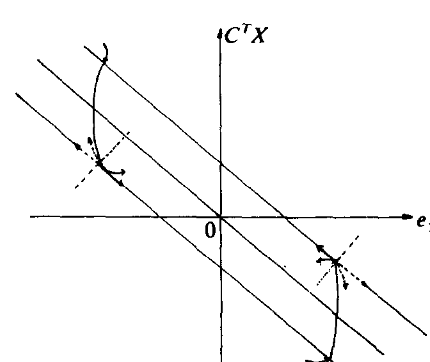

Fig. 2. Motion along the switching surface.

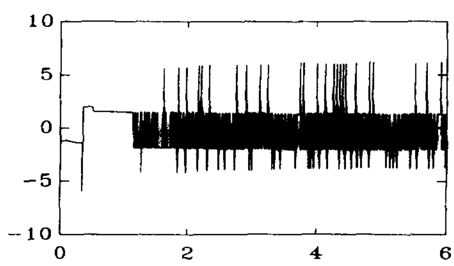

Fig. 3. Control signal $u(t)$.

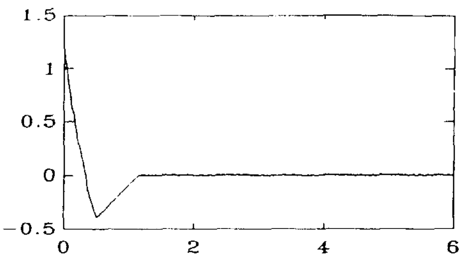

Fig. 5. Auxiliary error $e_1(t)$.

the region Ω, the VSAC with SFS will reduce $|c^T x(t)| + |e_1(t)|$ as deduced before. Thus the stability of the overall system is proved.

We should like to emphasize the following points: the control law defined by (31) is used to drive $x(t)$ and $e_1(t)$ into the region Ω in a finite time interval. Once $x(t)$ and $e_1(t)$ have been forced into the region Ω, they will stay in it. Inside Ω, all the signals are made bounded and small by the VSAC with SFS.

We can summarize the above analysis in the following theorem.

Theorem 4.1. For the system (1), if the assumptions (A1)–(A6) are satisfied, the switching surface S is selected according to (29), the control laws are defined by (31) in the region $|S| > \delta$ and by (39) in the region $|S| \le \delta$ and the logic switching function $\phi(t)$ is defined as in Section 3 then there exists a positive μ^* such that, for $|\mu| < \mu^*$, all the signals in the system are globally bounded and the system is globally stable.

5. *Simulation results*

In this section, we shall present some simulation results to illustrate the effectiveness of the variable structure robust adaptive control scheme given in Section 3. The plant to be controlled is expressed by

$$y_0(t) = p_0(s)[1 + \mu\Delta_1(s)]u(t) + \mu\Delta_2(s)u(t), \tag{53}$$

where

$$p_0(s) = \frac{(s-2)(s+1)}{(s+5)(s+3)(s-1)}, \quad \Delta_1(s) = \frac{1}{s+10},$$

$$\Delta_2(s) = \frac{1}{(s+3)(s+10)}, \quad \mu = 0.02.$$

The reference model is chosen to be

$$y_m(t) = \frac{1}{(s+1)^2(s+2)} r(t), \quad r(t) = 3\cos 5t.$$

In the control law (31) and (39), let $c^T = [1\ 2\ 1]$, $k_1 = 5$, $k_2 = 2$, $\delta = 0.4$, $\alpha = 2$, $\bar{\mu} = 0.05$ and $\sigma = 0.8$. In the adaptation rules (21), $\Gamma_1 = \Gamma_2 = I$ and $\gamma_3 = 1$. The simulation results are shown in Figs 3–5.

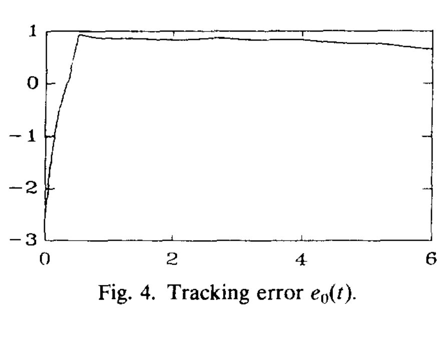

Fig. 4. Tracking error $e_0(t)$.

6. *Conclusions*

In this paper, we have presented a new variable structure robust model reference adaptive control scheme for linear time-invariant single-input, single-output systems when the relative degree of the modeled part of the plant is equal to or greater than one. Both additive and multiplicative unmodeled dynamics have been taken into consideration. Here the restriction of the minimum-phase condition is not necessary. The main idea of our present scheme is that the variable structure logic control and the sign-following system are introduced into the design of the control system. The algorithm presented in this paper provides an effective solution to a long-standing problem in which the modeled part of the plant is of non-minimum phase. The boundedness of all signals in the closed-loop system is guaranteed, and global stability and convergence are established. Though the control law proposed in this paper is for the case where the relative degree of modeled part of the plant is equal to one, similar results can be obtained for the more general case.

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